ÉCOLE POLYTECHNIQUE
UNIVERSITÉ PARIS-SACLAY

CONCOURS D'ADMISSION 2016 – FILIERE UNIVERSITAIRE INTERNATIONALE

SESSION PRINTEMPS 2016

PHYSICS

(Duration : 2 hours)

The three problems are independent. If you are not able to solve a question, we advise you to assume the result of that question and to move to the next one. The different questions are, to a large extent, independent of each other.

The use of electronic calculators is forbidden.

I. Molecular sizes

The aim of this problem is to find the size of the molecules of a liquid from the latent heat of vaporization (L) and the surface tension (γ) of the pure liquid.

Let M be the molar mass of the liquid and ε the interaction energy of one molecule with all the other molecules in the liquid. The molecules are assumed to be spherical with a diameter d. Their interaction is long ranged with a power-law decay :

$$U(r) = \frac{A}{r^n}$$

whenever $r > d$.

1) For which values of n is the energy ε well-defined? (hint: express the interaction energy of one molecule with all the other molecules contained in a macroscopic volume).

In the following, the molecules are assumed to be at the nodes of a simple cubic lattice, with a mesh size equal to d.

2) Express the mass density ρ of the liquid as a function of the molar mass M, the molecule diameter d and Avogadro constant N_A.

3) Let L be the latent heat of vaporization of the liquid (equal to the amount of heat required to transform 1 kg of liquid into vapor). Express L as a function of ε, M and N_A.

4) The surface tension γ of the liquid is defined as follows : when the open surface of the liquid (the surface separating the liquid and the air) is increased by an amount of dA, then the energy of the liquid increases accordingly by the amount :

$$dU = \gamma dA$$

5) Express γ as a function of ε and d.

6) Infer from this result that :

$$d = \frac{2\gamma}{\rho L}$$

7) Experimental data are given in the following table :

	ρ g/cm³	γ 10^{-3}N/m	L kJ/kg
water	1	73	2265
mercury	13,5	450	295
octane	0,703	22	298
glycerin	1,26	63	974

Find the diameters of the corresponding molecules. Conclude.

II. Gas leakage modeling

Notations and reminders :

(i) Boltzmann constant : k_B

(ii) gas particle density: n (number of molecules per unit volume)

(iii) gas molecular mass : m (mass of one molecule of the gas)

(iv) $\gamma = \frac{C_p}{C_v} = 1.4$ for a diatomic gas, with C_v (resp. C_p) the mass heat capacity at constant volume (resp. constant pressure).

(v) Sound speed in a perfect gas : $c = \sqrt{\gamma \frac{k_B T}{m}}$

(vi) vitesse moyenne des molécules dans un gaz parfait : $\bar{v} = \sqrt{\frac{8 k_B T}{\pi m}}$

A vessel, surrounded by vacuum, contains a perfect gas at pressure P and temperature T. At time t=0, a hole of diameter D is bored through the vessel wall.

1) The mean free path of the molecules in the gas is denoted λ. What is the physical meaning of the mean free path ? The molecules of the gas are assumed to be hard

spheres of diameter d and the particle volume density is n. Then the mean free path can be written as :

$$\lambda \approx \frac{1}{nd^2}$$

Express n as a function of P and T.

In the following, the thickness L of the vessel wall is assumed to be equal to λ.

2) Let us first assume that D < λ (« Knudsen regime»). Estimate the mass flow rate Q_m of the leaking gas. The gas pressure and temperature will be considered to be invariant in the vessel. It may further be assumed that one sixth of all the molecules in the vicinity of the hole have the same velocity of norm $\bar{v}$ pointing outwards in the normal direction to the wall. Justify this assumption.
How does the mass flow depend on D ?

From now on there is also a perfect gas outside the vessel. Both gases inside and outside the vessel are made of the same molecules, and are at the same temperature T ; the gas inside is at pressure P + ΔP and the gas outside at pressure P.

3) What is the new leak mass flow Q'_m?

4) Let us then assume that λ < D (« Poiseuille regime »). The gas can now be considered as a viscous fluid. If D is smaller than a critical value D_{max}, which will be specified in the next question, the gas flow is laminar through the pore (cylindric hole of diameter D and length L). In this case, the resistance to flow R_H, defined as :

$$R_H = \frac{\Delta P}{Q_v}$$

where ΔP is the excess of pressure inside the vessel (with respect to outside) and Q_v is the gas volume flow rate, can be written as :

$$R_H = \frac{128\eta L}{\pi D^4}$$

and the dynamic viscosity η of a perfect gas is :

$$\eta = \frac{1}{3} nm\bar{v}\lambda$$

Express the mass flow rate of the leaking gas as a function of ΔP, $\bar{v}$, n, d, L and D. How does it vary with D ?

5) It is reminded that the flow is laminar as long as Reynolds number

$$Re = \frac{< v > D}{\eta}$$

is smaller than 2000. Here $< v >$ is the velocity of the gas flow averaged over the pore cross-section. Compute $< v >$ and infer D_{max} from it.

6) Let us finally assume that $D > D_{max}$ (« Bernoulli regime »). It is reminded that the first law of thermodynamics for open systems can be written :

$$h + \frac{1}{2}v^2 = const$$

where h is the enthalpy per unit mass of gas and v the velocity of the gas flow, assumed to be uniform over the pore cross-section. Moreover $h = C_p T$ up to an arbitrary additive constant.
The flow is fast enough to be adiabatic and slow enough to be quasistatic. What is the physical meaning of these assumptions ?
Then according to Laplace law : $P^{1-\gamma}T^{\gamma} = const$. Prove that the velocity v of the gas flow satisfies the following equation:

$$1 + \frac{\gamma - 1}{2}\frac{v^2}{c^2} = \left(\frac{P + \Delta P}{P}\right)^{\frac{\gamma - 1}{\gamma}}$$

Infer from it the mass flow rate of the leaking gas. How does it depend on D ? For which values of ΔP does the sudden opening of a hole in the vessel wall result in a bang?

7) Plot the shape of the functional dependence of the mass flow rate Q_m on the hole diameter D in the most general case. Specify the values of D that correspond to the transition from Knudsen to Poiseuille regime on the one hand, and Poiseuille to Bernoulli regime on the other.

III. A charged particle moving in magnetic and electric fields

A point particle of mass m and charge q moves in a region where both a uniform electrostatic field $\vec{E}$ and a uniform magnetostatic field $\vec{B}$ are present. $\vec{E}$ and $\vec{B}$ are assumed to be orthogonal (see Figure). All relativistic effects are neglected in this problem.

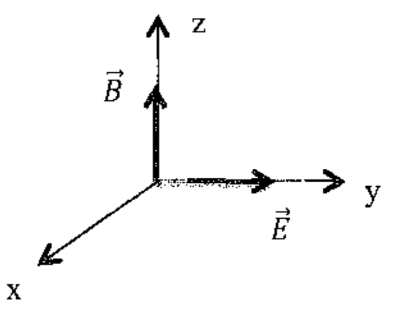

Figure : relative disposition of the $\vec{E}$ et $\vec{B}$ fields

1) Write the equation of motion of the particle.

2) Solve this equation for the particle velocity in the most general case. It is assumed that the initial velocity (velocity at time t=0), named $\vec{V}$, is normal to $\vec{B}$.

3) Express the time-average velocity $\vec{U}$ as a function of $\vec{E}$ et $\vec{B}$. Show that this result has an obvious interpretation in the reference frame moving at $\vec{U}$ relative to the original frame.

4) Give a physical description of the particle motion and plot the various trajectories depending on the ratio $\frac{VB}{E}$ (reminder : V is the initial velocity of the particle). What kind of trajectory is obtained for $V = 0$?

5) What is the condition on $\vec{E}$ and $\vec{B}$ that allows us to neglect relativistic effects ?